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ZERO MEMORY DETECTION OF RANDOM SIGNALS IN PHI-MIXING NOISE.(U)

1979 D R HALVERSON, G L WISE AFOSR-76-3062

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ZERO MEMORY DETECTION OF RANDOM SIGNALS IN ϕ-MIXING NOISE

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ABSTRACT

Design of detectors for ϕ-mixing signals in ϕ-mixing noise is considered, where a large degree of dependency may also occur between the signal and noise. Applying the criterion of asymptotic relative efficiency, it is shown that this design reduces to the solution of an integral equation in which knowledge of only the second-order statistics of the random processes involved is required. From this is may be seen that if the signal is independent of the noise and has nonzero mean, the optimal detector is the same as in the constant known signal case.

I. Introduction

A longstanding area of both practical and theoretical importance has been the detection of signals in corrupting noise. Because of modern high speed sampling, the presence of a dependent noise process is to be anticipated. Neyman-Pearson optimal techniques [1] are tractable only in cases where the appropriate multivariate distributions are known. Since in non-Gaussian situations these distributions are often not known, it has frequently been found fruitful to adopt an alternative fidelity criterion, commonly the asymptotic relative efficiency (ARE) criterion, which is especially appropriate in the weak signal situation. Because continuous time detection is often intractable in the non-Gaussian case, current efforts are directed toward discrete time detection. Results in this area have been obtained recently by Poor and Thomas [2,3] for the case of memoryless detection of a known constant signal in additive m-dependent noise; we have shown [4,5] how these results may be extended to a large class of ϕ-mixing noises, thus allowing the employment of noise models which more accurately reflect physical reality. The latter results guarantee performance (as measured by the ARE criterion) at least as good as that of the optimal detector [2,3] designed under the assumption of m-dependent noise (which may be taken to include "white noise" as a special case).

The class of processes used to model the noise in the above work may be seen to be quite general; however, the assumption of a constant known signal is in many cases overly restrictive. Instead of such an assumption, we might wish to model the signal as a random process. To set the problem in its greatest generality, we should allow dependency between signal samples (and thus employ ϕ-mixing models for both signal and noise), and we should also allow some degree of dependency between signal and noise. We are thus led to the problem of the design of the optimal detector in this general random signal situation.

II. Preliminaries

Let $\{X_i; i=1,2,\ldots\}$ be a strictly stationary sequence of random variables. For $a \le b$, define $M(a,b)=\sigma\{X_a, X_{a+1},\ldots,X_b\}$, the σ-algebra generated by the indicated random variables, where a and b may take on extended real values. Then $\{X_i; i=1,2,\ldots\}$ is symmetrically ϕ-mixing if there exists a nonnegative sequence $\{\phi_i; i=1,2,\ldots\}$ with $\phi_i \to 0$ such that for each k, $1 \le k < \infty$, and for each $i \ge 1$, $E_1 \in M(1,k)$ and $E_2 \in M(k+i,\infty)$ together imply

$$|P(E_1 \cap E_2) - P(E_1)\, P(E_2)| \le \phi_i \min\{P(E_1), P(E_2)\}\ .$$

We will consider detection of a symmetrically ϕ-mixing signal $\{S_i; i=1,2,\ldots\}$, where $0<E\{[S_i]^2\}<\infty$, in additive symmetrically ϕ-mixing noise $\{N_i; i=1,2,\ldots\}$, where we observe realizations $\{y_i; i=1,2,\ldots,n\}$ of the process $\{Y_i; i=1,2,\ldots,n\}$. In order to apply the ARE fidelity criterion, this will amount to a choice between the two hypotheses

$$H_0 : Y_i = N_i \ ;\ i=1,2,\ldots,n$$

$$H_1 : Y_i = N_i + \theta S_i \ ;\ i=1,2,\ldots,n$$

where θ is a parameter which will be allowed to approach zero at the proper rate, thus yielding the asymptotic limit. Throughout the discussion we will assume that both the noise and signal processes possess (possibly different) ϕ-representatives which satisfy

$$\sum_{i=1}^{\infty} \phi_i^{1/2} < \infty\ .$$

Any such symmetrically ϕ-mixing process will be called acceptable. For convenience we assume the existence of densities $f_j(\cdot,\cdot)$ of N_k and N_{k+j}, $f(\cdot)$ of N_1, $f(\cdot,\cdot)$ of N_k and S_k, where the latter is assumed to be independent of k. We also assume

$$K_n^*(x,y) \triangleq \sum_{j=1}^{n} [f_j(x,y)+f_j(y,x)] \big/ \sqrt{f(x)f(y)}$$

is square integrable for all n, and that $f(\cdot)$ is strictly positive on the real line. We assume in addition that

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$$\int y^2 \frac{\partial^2}{\partial x^2} f(x,y)\, dy \,/\, \sqrt{f(x)}$$

and

$$\int y \frac{\partial}{\partial x} f(x,y)\, dy \,/\, \sqrt{f(x)}$$

are square integrable (all integrals, unless indicated otherwise, are taken over the entire real line). Note that if the signal and noise are independent, the latter condition is equivalent to the assumption of finite Fisher's information number contained in [2,3] and [4,5]. We also assume that

$$\lim_{\theta\to 0} \int f(x-\theta y, y)\, dy = f(x).$$

As in [2,3] and [4,5], we will optimize over the class of optimal memoryless detectors designed under a "white noise" assumption, i.e. where a test statistic $T_g(y) = \sum_{i=1}^{n} g(y_i)$ is compared to a threshold. Specifying g will therefore be of prime concern.

We will restrict the class $\mathcal{G}$ of nonlinearities g to include those measurable real valued functions for which we can find $\theta_1 > 0$ such that the random variable $g(N_1 + \theta S_1)$ is second-order for all $\theta \in [0,\theta_1]$, and such that the following mild regularity conditions hold, where $E_\theta(\cdot)$ denotes expectation computed under H_1 with parameter θ (by proper choice of the threshold, we assume without loss of generality that the random variables $g(N_i)$ are zero mean):

(a) $\int g(x) f'(x)dx \neq 0$

if the signal and noise processes are independent

(b) $$\lim_{n\to\infty} \frac{\left[\frac{\partial}{\partial\theta} E_\theta\{T_g(Y)\}\big|_{\theta=0}\right]^2}{nE_0\left\{[T_g(Y)]^2\right\}} \triangleq \eta(g) > 0$$

if $\iint y g(x) \frac{\partial}{\partial x} f(x,y)\, dxdy \neq 0$, or

(b') $$\lim_{n\to\infty} \frac{\left[\frac{\partial^2}{\partial\theta^2} E_\theta\{T_g(Y)\}\big|_{\theta=0}\right]^2}{nE_0\left\{[T_g(Y)]^2\right\}} \triangleq \eta(g) > 0$$

if $\iint y g(x) \frac{\partial}{\partial x} f(x,y)\, dxdy = 0$

(c) $$-\infty < \lim_{n\to\infty} \frac{\partial}{\partial\theta} E\{g(N_1+\theta S_1)\}\big|_{\theta=k_1/\sqrt{n}} = \frac{\partial}{\partial\theta} E\{g(N_1+\theta S_1)\}\big|_{\theta=0} < \infty$$

for some constant $k_1 > 0$

if $\iint y g(x) \frac{\partial}{\partial x} f(x,y)\, dxdy \neq 0$, or

(c') $$-\infty < \lim_{n\to\infty} \frac{\partial^2}{\partial\theta^2} E\{g(N_1+\theta S_1)\}\big|_{\theta=k_2/n^{1/4}} = \frac{\partial^2}{\partial\theta^2} E\{g(N_1+\theta S_1)\}\big|_{\theta=0} < \infty$$

for some constant $k_2 > 0$

if $\iint y g(x) \frac{\partial}{\partial x} f(x,y)\, dxdy = 0$

(d) $$\left| \frac{\partial}{\partial\theta} E\left\{[g(N_1+\theta S_1)]^2\right\}\big|_{\theta=0} \right| < \infty$$

(e) $$\frac{\partial}{\partial\theta} \iint g(x)\, f(x-\theta y, y)\, dxdy \Big|_{\theta=0} = \iint \frac{\partial}{\partial\theta} g(x)\, f(x-\theta y,y)\Big|_{\theta=0} dxdy$$

if $\iint y g(x) \frac{\partial}{\partial x} f(x,y)\, dxdy \neq 0$, or

(e') $$\frac{\partial^2}{\partial\theta^2} \iint g(x)\, f(x-\theta y, y)\, dxdy \Big|_{\theta=0} = \iint \frac{\partial^2}{\partial\theta^2} g(x)\, f(x-\theta y,y)\Big|_{\theta=0} dxdy$$

if $\iint y g(x) \frac{\partial}{\partial x} f(x,y)\, dxdy = 0$

(f) $$\sigma_0^2(g) \triangleq E\{g(N_1)^2\} + 2\sum_{j=1}^{\infty} E\{g(N_1)g(N_{j+1})\} > 0.$$

The restrictions on the densities $f_j(\cdot,\cdot)$, $f(\cdot)$, and $f(\cdot,\cdot)$ and the class $\mathcal{G}$ are what might be expected when compared to those of [2,3] and [4,5] for the constant known signal case. Properties (a)-(c') are assumptions conventionally imposed for application of the Pitman-Noether theorem [6], whereas (d)-(e') are exceedingly mild restrictions. For a large class of processes, including all of the examples of [3], property (f) is satisfied and may therefore be ignored.

III. Development

The tractability of the ARE approach is derived chiefly from an appeal to central limit theorem results, which in our case arise out of the imposition of a mixing condition on the dependency structure. The following lemma, for the case of independent signal and noise, is the first step toward obtaining the appropriate mixing condition under H_1:

Lemma 1: If $\{N_i; i=1,2,\ldots\}$ and $\{S_i; i=1,2,\ldots\}$ are acceptable and independent processes, then the process $N_1, S_1, N_2, S_2, \ldots$ is acceptable.

Proof: Let $E_1 \in \sigma\{N_1, S_1, \ldots, N_k, S_k\}$ and $E_2 \in \sigma\{N_{k+1}, S_{k+1}, \ldots\} \triangleq \mathfrak{F}$. In a manner similar to that of [7], we conclude from [8,p.37] that (for fixed i,k) $E\left\{ E\{I_{E_2} | \mathfrak{F}_{i+1}\} \big| \mathfrak{F}_i \right\} = E\{I_{E_2} | \mathfrak{F}_i\}$, where $\mathfrak{F}_j \triangleq \sigma\{N_{k+1}, S_{k+1}, \ldots, N_{k+j}, S_{k+j}\}$, and hence $\{E\{I_{E_2} | \mathfrak{F}_j\}, \mathfrak{F}_j, j \geq i\}$ is a martingale. It follows

from [8,p.332] that $E\{I_{E_2}|\sigma_j\} \to E\{I_{E_2}|\sigma\} = I_{E_2}$ wp1. Since $E\{|E\{I_{E_2}|\sigma_j\}|\} \le 1$, it follows from the martingale convergence theorem [8, p.319] due to Doob that $E\{I_{E_2}|\sigma_j\} \to I_{E_2}$ in $L_1(\mu)$, where μ is the measure induced by $N_{k+i}, S_{k+i}, \ldots$. Therefore, it follows from [8, p.603] that there exist measurable $h_j: \mathbb{R}^{2(j-i+1)} \to \mathbb{R}$ satisfying $h_j(N_{k+i}, S_{k+i}, \ldots, N_{k+j}, S_{k+j}) = E\{I_{E_2}|\sigma_j\}$. Note also that $|h_j| \le 1$ for all values of the argument, and $h_j(N_{k+i}, S_{k+i}, \ldots, N_{k+j}, S_{k+j}) \to I_{E_2}$ in $L_1(\mu)$. It also follows from [8, p.603] that there exist measurable $w: \mathbb{R}^{2k} \to \mathbb{R}$ such that $w(N_1, S_1, \ldots, N_k, S_k) = I_{E_1}$. We will now introduce some notation which will simplify the development of the proof (in the following i and k are fixed):

$$h_j \triangleq h_j(N_{k+i}, S_{k+i}, \ldots, N_{k+j}, S_{k+j})$$

$$\tilde{w} \triangleq w(x_1, y_1, \ldots, x_k, y_k)$$

$$\tilde{h}_j \triangleq h_j(x_{k+i}, y_{k+i}, \ldots, x_{k+j}, y_{k+j})$$

$$dF_N = dF_N(x_1, \ldots, x_k)$$

where $F_N(\cdots)$ is the distribution function of $N_1, \ldots, N_k$,

$$dF_N(j) = dF_{N,j}(x_{k+i}, \ldots, x_{k+j})$$

where $F_{N,j}(\cdots)$ is the distribution function of $N_{k+i}, \ldots, N_{k+j}$,

$$dF_S = dF_S(y_1, \ldots, y_k)$$

where $F_S(\cdots)$ is the distribution function of $S_1, \ldots, S_k$,

$$dF_S(j) = dF_{S,j}(y_{k+i}, \ldots, y_{k+j})$$

where $F_{S,j}(\cdots)$ is the distribution function of $S_{k+i}, \ldots, S_{k+j}$,

$$dF_N(k,j) = dF_{N,k,j}(x_1, \ldots, x_k, x_{k+i}, \ldots, x_{k+j})$$

where $F_{N,k,j}(\cdots)$ is the distribution function of $N_1, \ldots, N_k, N_{k+i}, \ldots, N_{k+j}$,

$$dF_S(k,j) = dF_{S,k,j}(y_1, \ldots, y_k, y_{k+i}, \ldots, y_{k+j})$$

where $F_{S,k,j}(\cdots)$ is the distribution function of $S_1, \ldots, S_k, S_{k+i}, \ldots, S_{k+j}$,

$$dF_{N,S} = dF_{N,S}(x_1, y_1, \ldots, x_k, y_k)$$

where $F_{N,S}(\cdots)$ is the distribution function of $N_1, S_1, \ldots, N_k, S_k$,

$$dF_{N,S}(j) = dF_{N,S,j}(x_{k+i}, y_{k+i}, \ldots, x_{k+j}, y_{k+j})$$

where $F_{N,S,j}(\cdots)$ is the distribution function of $N_{k+i}, S_{k+i}, \ldots, N_{k+j}, S_{k+j}$, and

$$dF_{N,S}(k,j) =$$
$$dF_{N,S,k,j}(x_1, y_1, \ldots, x_k, y_k, x_{k+i}, y_{k+i}, \ldots x_{k+j}, y_{k+j})$$

where $F_{N,S,k,j}(\cdots)$ is the distribution function of $N_1, S_1, \ldots, N_k, S_k, N_{k+i}, S_{k+i}, \ldots, N_{k+j}, S_{k+j}$.

We then have, for $\epsilon > 0$ and large j,

$$|P(E_1 \cap E_2) - E\{I_{E_1} h_j\}| \le \text{P-ess sup } |I_{E_1}| \cdot E\{|I_{E_2} - h_j|\} < \epsilon \tag{1}$$

and

$$|P(E_1)P(E_2) - E\{I_{E_1}\}E\{h_j\}| \le \text{P-ess sup } |I_{E_1}| \cdot E\{|I_{E_2} - h_j|\} < \epsilon . \tag{2}$$

Now

$$E\{I_{E_1} h_j\} - E\{I_{E_1}\} E\{h_j\} =$$
$$\int \tilde{w}\, \tilde{h}_j\, dF_{N,S}(k,j) - \int \tilde{w}\, dF_{N,S} \int \tilde{h}_j\, dF_{N,S}(j) =$$
$$\iint \tilde{w}\, \tilde{h}_j\, dF_N(k,j)\, dF_S(k,j) -$$
$$\iiiint \tilde{w}\, \tilde{h}_j\, dF_N\, dF_N(j)\, dF_S\, dF_S(j) ,$$

because of the independence of signal and noise; and thus

$$\begin{aligned} &|E\{I_{E_1} h_j\} - E\{I_{E_1}\} E\{h_j\}| \le \\ &\left|\int \left\{ \int \tilde{w}\, \tilde{h}_j\, dF_N(k,j) - \int \tilde{w}\, dF_N \int \tilde{h}_j\, dF_N(j) \right\} dF_S(k,j)\right| \\ &+ \left| \iint w\, dF_N \int \tilde{h}_j\, dF_N(j)\, dF_S(k,j) \right. \\ &\left. - \iiiint \tilde{w}\, \tilde{h}_j\, dF_N\, dF_N(j)\, dF_S\, dF_S(j) \right| . \end{aligned} \tag{3}$$

Now [9, p.170] implies that the first summand on the right hand side of (3) can be upper bounded by

$$\int 2\phi_i\, E\{|w(N_1, y_1, \ldots, N_k, y_k)|\} \cdot$$
$$(\text{P-ess sup } |h_j|)\, dF_S(k,j)$$
$$\le 2\phi_i E\{w(N_1, S_1, \ldots, N_k, S_k)\}$$

$= 2\phi_i P(E_1)$ where $\{\phi_i; i=1,2,\ldots\}$ is a ϕ-representation for $\{N_i; i=1,2,\ldots\}$. In a similar manner, we find that the second summand on the right hand side of (3)

can be upper bounded by $2\psi_i\, E\{w(N_1,S_1,\ldots,N_k,S_k)\} = 2\psi_i\, P(E_1)$, where $\{\psi_i;i=1,2,\ldots\}$ is a ϕ-representation for $\{S_i;i=1,2,\ldots,\}$. These bounds thus imply $|E\{I_{E_1}h_j\}-E\{I_{E_1}\}E\{h_j\}| \le 2(\phi_i + \psi_i)\, P(E_1)$, and hence it follows from (1) and (2), by allowing $j \to \infty$, that

$$|P(E_1 \cap E_2) - P(E_1)P(E_2)| \le 2(\phi_i + \psi_i)\, P(E_1)\ .$$

An analogous argument in conjunction with a straightforward modification of [9, p. 170] (where $\phi_n^{1/s}$ replaces $\phi_n^{1/r}$) yields

$$|P(E_1 \cap E_2) - P(E_1)P(E_2)| \le 2(\phi_i + \psi_i)P(E_2)\ .$$

Therefore, we conclude that $N_1,S_1,N_2,S_2,\ldots,$ is symmetrically ϕ-mixing with ϕ-representation $\{2(\phi_i + \psi_i);i=1,2,\ldots\}$, and is therefore acceptable.

QED

In the case where there exists dependency between the signal and noise the situation is more involved. For many cases of engineering interest, where the noise is dependent on a finite "window" of the signal, such as the signal-dependent noise induced through reverberation effects, we can obtain the desired result. The extension of Lemma 1 to this signal-dependent noise case is given by the following:

<u>Lemma 2</u>: Suppose $\{S_i;i=1,2,\ldots\}$ is acceptable, and for a fixed nonnegative integer m, $N_i=G(X_i,Z_i)$ for $i=1,2,\ldots$ where X_i is $\sigma\{S_{i-m},\ldots,S_{i+m}\}$ measurable, $G\colon \mathbb{R}^2 \to \mathbb{R}$ is measurable, and $\{Z_i;i=1,2,\ldots\}$ is accetable and independent of $\{S_i;i=1,2,\ldots\}$ (we let $S_{i-m} \triangleq S_1$ for $i \le m$). Then $N_1,S_1,N_2,S_2,\ldots$ is acceptable.

<u>Proof</u>: We infer from Lemma 1 that $Z_1,S_1,Z_2,S_2,\ldots$ is acceptable. There exist [8, p.603] measurable functions q_i with $X_i=q_i(S_{i-m},\ldots,S_{i+m})$, where S_1 appears only once if $i \le m$. For each integer $i > 2m$, define $\tilde{\mathcal{F}}_i\colon \mathbb{R}^{4m+2} \to \mathbb{R}$ by $\tilde{\mathcal{F}}_i(x_1,\ldots,x_{4m+2}) =$

$$\begin{cases} G[q_{(i+1)/2}(x_2,x_4,\ldots,x_{4m+2}),x_{2m+1}] & \text{if } i \text{ is odd} \\ x_{2m+1} & \text{if } i \text{ is even} \end{cases}$$

Define a process $\{U_i;i=1,2,\ldots\}$ by

$$U_i = \begin{cases} Z_{(i+1)/2} & \text{if } i \text{ is odd} \\ S_{i/2} & \text{if } i \text{ is even} \end{cases}\ .$$

Note that $\{U_i;i=1,2,\ldots\}$ is just $Z_1,S_1,Z_2,S_2,\ldots$ and is, therefore, acceptable. We then define, for $i > 2m$, $V_i = \tilde{\mathcal{F}}_i(U_{i-2m},\ldots,U_{i+2m+1})$. If $1 \le i \le 2m$, define $\tilde{\mathcal{F}}_i\colon \mathbb{R}^{i+2m+1} \to \mathbb{R}$ by

$$\tilde{\mathcal{F}}_i(x_1,\ldots,x_{i+2m+1}) = \begin{cases} G[q_{(i+1)/2}(x_2,x_4,\ldots,x_{i+2m+1}),x_i] & \text{if } i \text{ is odd} \\ x_i & \text{if } i \text{ is even} \end{cases},$$

and let $V_i = \tilde{\mathcal{F}}_i(U_1,\ldots,U_{i+2m+1})$. We thus obtain, for $i=1,2,\ldots$

$$V_i = \begin{cases} N_{(i+1)/2} & \text{if } i \text{ is odd} \\ S_{i/2} & \text{if } i \text{ is even} \end{cases}\ ;$$

hence $N_1,S_1,N_2,S_2,\ldots$ arises via the time varying finite memory transformation $\tilde{\mathcal{F}}_i$ on the acceptable process $\{U_i;i=1,2,\ldots\}$. The desired result thus follows from a straightforward modification of the proof of Proposition 7 of [5].

QED

We now are in a position to obtain a significant result pertinent to the detection context:

<u>Theorem 1</u>: Suppose $\{\mathcal{F}_i\colon \mathbb{R}^{2p+2} \to \mathbb{R};i=1,2,\ldots\}$ is a family of measurable functions where p is a fixed nonnegative integer. Then under the hypothesis of Lemma 1 or Lemma 2, we have that $\{\mathcal{F}_i(N_i,S_i,\ldots,N_{i+p},S_{i+p});i=1,2,\ldots\}$ is acceptable.

<u>Proof</u>: This follows as a consequence of Lemma 2 and a straightforward modification of Proposition 7 of [5].

QED

A result of interest now follows as a direct corollary:

<u>Corollary</u>: Under the hypothesis of Lemma 1 and Lemma 2, $\{Y_i;i=1,2,\ldots\}$ is symmetrically ϕ-mixing under H_1, with ϕ-representation independent of θ and given by $\Phi_1=1$, $\Phi_i=2(\phi_{i-1} + \psi_{i-1})$ for $i=1,2,3,\ldots$ under the hypothesis of Lemma 1, and $\Phi_1=\Phi_2=\ldots=\Phi_{4m+2}=1$, $\Phi_i=2(\pi_{i-4m-2} + \psi_{i-4m-2})$ for $i=4m+3,4m+4,\ldots$ under the hypothesis of Lemma 2, where $\{\phi_i\}_i,\{\psi_i\}_i,\{\pi_i\}_i$ are associated with $\{N_i\}_i$ (under the hypothesis of Lemma 1), $\{S_i\}_i$, and $\{Z_i\}_i$, respectively.

<u>Proof</u>: This follows from Theorem 1, the proofs of Lemma 1 and Lemma 2, and a straightforward modification of the proof of Proposition 7 of [5].

QED

We now can obtain the result which will allow employment of the Pitman-Noether Theorem [6].

<u>Theorem 2</u>: Suppose $\theta_n \in \mathbb{R}$ with $\theta_n \to 0$, and $g \in \mathcal{G}$. Let $T_{n,\theta} = \frac{T_g}{\sqrt{n}}$ under H_1 with parameter θ_n, where the noise and signal processes satisfy the hypoth-

esis of Lemma 1 or Lemma 2. Then $\sigma_0^2(g)$ converges absolutely, $\sigma_{n,\theta}^2 \triangleq E\{(T_{n,\theta} - E\{T_{n,g}\})^2\} \to \sigma_0^2(g)$, and $\frac{T_{n,\theta} - E\{T_{n,\theta}\}}{\sigma_{n,\theta}} \xrightarrow{\mathcal{D}} N(0,1)$.

Proof: Note that $\{N_i; i=1,2,\ldots\}$ is acceptable under the hypothesis of Lemma 1 and Lemma 2, the latter following as a result of Theorem 1. It therefore follows from [9, p.184] that $\sigma_0^2(g)$ converges absolutely. Letting T_{g0} denote T_g under H_0, and applying Proposition 7 of [5] to $\{g(N_i); i=1,2,\ldots\}$, we conclude from the proof of Theorem 20.1 in [9, pp.184-190] that T_{g0}^2/n is uniformly integrable, and hence it follows from [9, pp.33,184] and assumption (f) that

$$\frac{E\{T_{g0}^2\}}{n\sigma_0^2(g)} \to 1. \tag{4}$$

Furthermore, from [9, p.170] and Theorem 1 we conclude

$$E\{(T_{n,\theta} - E\{T_{n,\theta}\} - T_{g0}/\sqrt{n})^2\}^{\frac{1}{2}} \le$$

$$(1 + 4 \sum_{i=1}^{\infty} \phi_i^{\frac{1}{2}})^{\frac{1}{2}} \left[E\{g(N_1 + \theta_n S_1) - g(N_1))^2\}^{\frac{1}{2}} + |E\{g(N_1 + \theta_n S_1)\}|\right], \text{where } \sum_{i=1}^{\infty} \phi_i^{\frac{1}{2}} < \infty .$$

Using a technique similar to that employed in [10] (or a result given in [11]), we conclude from assumptions (c) - (c') and (d) that

$$E\{(T_{n,\theta} - E\{T_{n,\theta}\} - T_{g0}/\sqrt{n})^2\} \to 0, \tag{5}$$

which when combined with (4) yields $\sigma_{n,\theta}^2 \to \sigma_0^2(g)$. The final result follows from (5) and [9, pp.25,184].

QED

We can now obtain the main result:

Theorem 3: Suppose that the hypothesis of Lemma 1 or Lemma 2 is satisfied, and $g \in \mathcal{G}$. Then g is optimal (in the sense of the ARE) if and only if g satisfies (up to a scale factor)

$$\text{(A)} \quad \sum_{j=1}^{\infty} \int [f_j(x,y) + f_j(y,x)]\, g(y)dy + f'(x) = -f(x)g(x)$$

if $\{N_i\}_{i=1}^{\infty}$ and $\{S_i\}_{i=1}^{\infty}$ are independent and $E\{S_1\} \neq 0$,

or

$$\text{(B)} \quad \sum_{j=1}^{\infty} \int [f_j(x,y) + f_j(y,x)]\, g(y)dy + f''(x) = -f(x)g(x)$$

if $\{N_i; i=1,2,\ldots\}$ and $\{S_i = i=1,2,\ldots\}$ are independent and $E\{S_1\} = 0$, where $f_j(\cdot,\cdot)$ is the joint density of N_1 and N_{j+1}, and f is the univariate density of N_1, or

$$\text{(C)} \sum_{j=1}^{\infty} \int [f_j(x,y) + f_j(y,x)]\, g(y)dy + \int y \frac{\partial}{\partial x} f(x,y)dy = -f(x)g(x), \text{ if } \iint yg(x) \frac{\partial}{\partial x} f(x,y)\, dxdy \neq 0, \text{ or}$$

$$\text{(D)} \sum_{j=1}^{\infty} \int [f_j(x,y) + f_j(y,x)]\, g(y)dy + \int y^2 \frac{\partial^2}{\partial x^2} f(x,y)dy = -f(x)g(x), \text{ if}$$

$\iint yg(x) \frac{\partial}{\partial x} f(x,y)dxdy = 0$, where $f(\cdot,\cdot)$ is the joint density of N_1 and S_1.

Proof: Referring to the Pitman-Noether Theorem [6], with $(m,\delta) = (1,\frac{1}{2})$ or $(m,\delta) = (2,\frac{1}{4})$, we note that assumptions (b)-(b') together with negative scaling of the threshold if necessary imply conditions A and B of the Pitman-Noether Theorem, whereas (c)-(c') imply the first part of condition C. We infer the second part of condition C from (4) and (5), and condition D' from Theorem 2. Consider now the proof of (C). We employ the Pitman-Noether Theorem with $\theta_n = k_1/\sqrt{n}$, and obtain an expression for the efficacy $\eta(g)$ given by $\eta(g) = [\frac{\partial}{\partial\theta} E_\theta\{g(Y_1)\}\big|_{\theta=0}]^2/\sigma_0^2(g)$.

Assumption (e) then implies $\frac{\partial}{\partial\theta} E_\theta\{g(Y_1)\}\big|_{\theta=0} =$

$$\frac{\partial}{\partial\theta} \iint g(x+\theta y) f(x,y)dxdy\Big|_{\theta=0} =$$

$-\iint yg(x) \frac{\partial}{\partial x} f(x,y)dxdy \neq 0$. Using the methods of Theorem 1 of [5] we obtain the desired result. To obtain (D), we note that $\frac{\partial}{\partial\theta} E_\theta\{g(Y_1)\}\big|_{\theta=0} = 0$, so we let $\theta_n = k_2/n^{\frac{1}{4}}$, and obtain $\eta(g) = \left[\frac{\partial^2}{\partial\theta^2} E_\theta\{g(Y_1)\}\Big|_{\theta=0}\right]^2 \Big/ \sigma_0^2(g)$. In this case, assumptions (b') and (e') imply

$$\frac{\partial^2}{\partial\theta^2} E_\theta\{g(Y_1)\}\Big|_{\theta=0} = \frac{\partial^2}{\partial\theta^2} \iint g(x+\theta y) f(x,y)dxdy\Big|_{\theta=0}$$

$$= -\iint \frac{\partial}{\partial\theta} yg(x) \frac{\partial}{\partial x} f(x-\theta y,y)\Big|_{\theta=0} dxdy =$$

$\iint y^2 g(x) \frac{\partial^2}{\partial x^2} f(x,y)dxdy \neq 0$, and we obtain (D) in the same way as (C). We derive (A) and (B) in a similar manner by employing assumption (a) and noting that $f(\cdot,\cdot)$ factors.

QED

Note that the methods of [4,5] may be employed to obtain the solution of the required integral

equation, which is of nonstandard form. The bounds of Proposition 5 and 6 of [5] may be obtained through the use of the corollary to Theorem 1.

IV. Conclusion

We have considered the design of the optimal detector for signal detection in corrupting noise, where both the signal and noise may be chosen from a large class of ϕ-mixing processes and may be dependent on each other. We have seen that this design reduces to the solution of an integral equation in which knowledge of only the second-order statistics of the random processes involved is required. In particular, if the signal is independent of the noise and has nonzero mean, the optimal detector is the same as in the constant known signal case.

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